performance over the 3.7–4.2-GHz band. The conditions which must be satisfied for a perfect match are

$$\left\{\frac{1}{Y^2}\sin^2\frac{\pi}{2}(1+\Delta)+\cos^2\frac{\pi}{2}(1+\Delta)\right\}G=1$$

$$\left\{\frac{1}{Y^2}\sin^2\frac{\pi}{2}(1+\Delta)+\cos^2\frac{\pi}{2}(1+\Delta)\right\}B(G)\Delta$$
$$=\sin\frac{\pi}{2}(1+\Delta)\cos\frac{\pi}{2}(1+\Delta)\left\{\frac{1}{Y}-Y\right\} \quad (12)$$

where Y is the transformer admittance level, G is the equivalent conductance, $B(G)$ is the susceptance slope parameter, and $\Delta = (\omega - \omega_0)/\omega_0$ with ω_0 being the circulator center frequency and ω the frequency for which the device should be matched. These conditions are obtained from (6) and (7) if the *ABCD* matrix elements of a single-step transformer are substituted. Notice that these equations remain unchanged if Δ is replaced by $-\Delta$. This means that there will be two frequencies of perfect match symmetrically located about the center frequency. If we choose $\Delta = 0.0447$ so as to have optimum performance over the 3.7–4.2 band, then (12) constitutes two equations in two unknowns Y and G. The dependence of B upon G introduces a complication in their solution. In practice, they were solved by assuming values for B and then solving for Y and G until the value of B obtained from the curve of Fig. 4 for the calculated value of G agreed with assumed value. The result for $\Delta = 0.0447$ was $B = 6.3$, $Y = 1.805$, and $G = 3.217$. The theoretical VSWR is less than 1.01 over the band, which obviously will not be obtainable in practice because of irreproducible connector mismatches. Transformer dielectrics with a dielectric constant giving an admittance level close to 1.80 were now inserted in each arm of the circulator. The resulting device had a performance better than 30 dB over the 3.7–4.2 band.

V. Conclusions

The equivalent or complex gyrator admittance of a standard stripline circulator junction has been measured with a computerized measurement system as a function of frequency and magnetic field. The results confirm that this admittance is that of a shunt resonator close to the first dielectric resonant frequency of the garnet disks. Several interesting features did emerge. The conductance initially increases linearly with magnetic field but then begins to saturate approaching a saturation value of 3.25. The susceptance slope parameter B of the resonator is a function of the magnetic field and decreases in the region where the conductance is saturating. The data were used to determine the matching transformer required to build a device with better than 30 dB return loss over the 3.7–4.2-GHz band.

Acknowledgment

The author wishes to thank D. Hagerman and J. Madden for diligent efforts in getting the computerized measurement system in operation.

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Dimensions of Microstrip Coupled Lines and Interdigital Structures

THOMAS A. MILLIGAN, MEMBER, IEEE

Abstract—**A method is presented for finding dimensions of coupled lines and interdigital structures on microstrip given the electrical properties. Both a graphical approach and computer approach using polynomial approximations are given. These results are within 1 percent of Bryant and Weiss' dimensions for coupled lines for most practical stripwidths and spacings. Experimental data for a 10-percent bandwidth microstrip interdigital filter are given.**

I. Introduction

The design of microwave filters and couplers on microstrip requires data on coupled lines and interdigital structures. The design of these structures has been done using tables or graphs generated from the work of Bryant and Weiss [1], Smith [2], or others. Until now there has been no method of designing microstrip interdigital structures. All of these methods start with dimensions and end with the electrical properties of the structure. This short paper gives a method of obtaining dimensions of the lines from the self and mutual capacitances. The method is presented in both a graphical form and polynomial approximations which can be programmed. Coupled lines with unequal linewidths and interdigital structures can be approximated.

The curves obtained here follow the same idea as Cristal's [5] method for coupled rods between ground planes. Cristal derived the curves by analyzing a periodic structure of equal rods and devised an approximation method to find interdigital structure dimensions with these curves. The same idea can be extended to microstrip with a few minor changes. It should be pointed out that this method can find stripwidths and spacings, but it does not handle the problem of finding the velocity of the waves on the lines. For N lines in an inhomogeneous structure such as microstrip, there are N possible normal modes with, in general, N different velocities. Interdigital and combline filters do not fit the normal modes, and it is a problem determining the proper length to make the lines. The proper resonator length of even the simpler coupled line filters is difficult to determine. The method was tried experimentally on a five-section interdigital filter and shows usable results even though the filter deviates from the theoretical response. Section II covers the derivation and use of the curves, Section III gives polynomial approximations of the curves and explains their use, and Section IV reports on an experimental filter.

Manuscript received October 13, 1975; revised August 23, 1976.
The author is with the Aerospace Division, Martin Marietta Corporation, Denver, CO 80201.

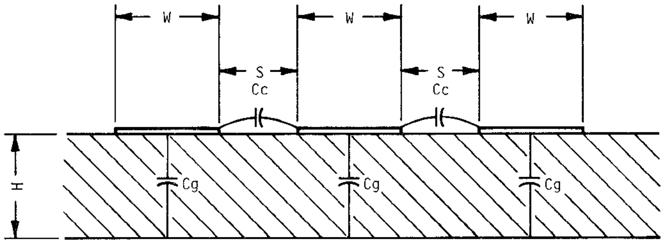

Fig. 1. Mutual and self-capacitances of periodic array of interdigitated microstrip conductors.

II. Derivation of the Curves

To analyze the coupled line structure on microstrip, a periodic structure of strips was assumed, as shown in Fig. 1. The parameters of this structure were analyzed by defining normalized capacitances per unit length and calculated by modifying the computer program MSTRIP of Bryant and Weiss [1].

The normalized capacitances of a pair of coupled lines with equal widths are defined as

$$C_{oe} = 50/Z_{oe} \quad C_{oo} = 50/Z_{oo} \tag{1}$$

where Z_{oe} and Z_{oo} are the even and odd mode impedances of the coupled pair of lines. This is identical to the normalization of Wenzel [3] for interdigital structures. The normalized capacitances are related to the normalization of Getsinger [4] for interdigital structures by

$$C/\varepsilon_0 = 7.534\, C_{\text{norm}} \tag{2}$$

for a 50-Ω system, where C_{norm} is the normalization of Wenzel and C/ε_0 is the normalization of Getsinger.

For a pair of equal width coupled lines, these normalized capacitances can be subdivided

$$C_{oe} = C_p + C_f + C_{fe} \quad C_{oo} = C_p + C_f + C_{fo} \tag{3}$$

where C_p is the parallel plate capacitance, C_f is the fringing capacitance on the uncoupled side, and C_{fe} and C_{fo} are the coupled even and odd mode fringing capacitances, respectively. The periodic structure can be described by C_g, the self-capacitance, and C_c, the mutual capacitance. These are related to the parallel plate capacitance and fringing capacitances by

$$C_g = C_p + 2C_{fe} \quad C_c = (C_{fo} - C_{fe})/2. \tag{4}$$

The normalized capacitance of an uncoupled strip can also be subdivided

$$C_0 = C_p + 2C_f \tag{5}$$

where C_p is the parallel plate capacitance and C_f is the fringing capacitance on one side of the strip.

The self-capacitance and mutual capacitance of the periodic structure was found by analyzing a pair of equal width coupled lines and an uncoupled strip of the same width. Combining (3) and (5), the self and mutual capacitances of the periodic structure are related to these by

$$C_g = 2C_{oe} - C_0 \quad C_c = (C_{oo} - C_{oe})/2 \tag{6}$$

where C_0 is the normalized capacitance of the uncoupled strip.

The program MSTRIP was modified to prepare the data. W/H was held constant and S/H increased exponentially to stress smaller values of S/H in the polynomial approximations. Figs. 2 and 3 present the data graphically for a dielectric constant of 9.6 which is identical in form to Cristal's [5] graphs for round rods between parallel ground planes. The procedure to use these

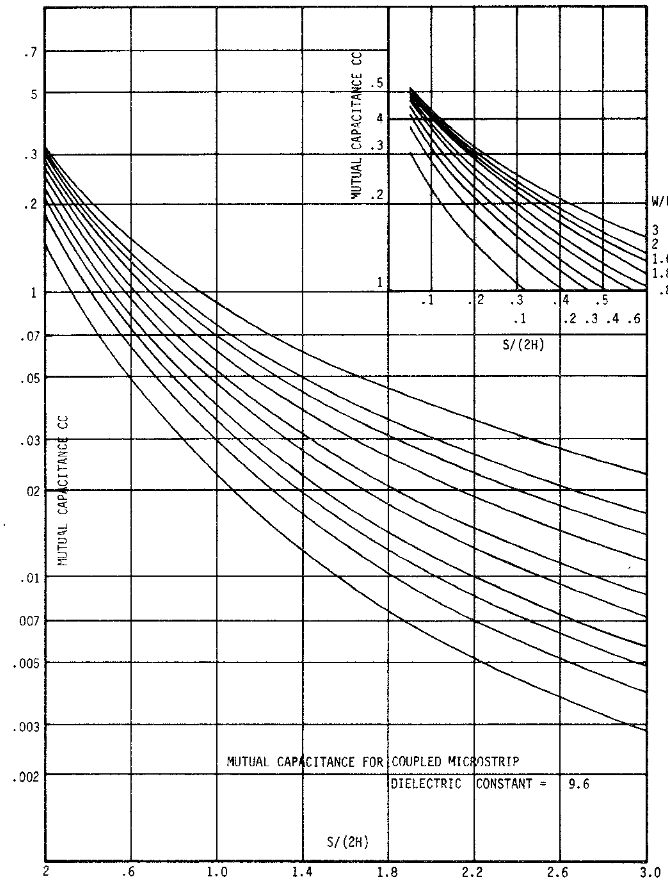

Fig. 2. Mutual capacitance for coupled microstrip dielectric constant of 9.6.

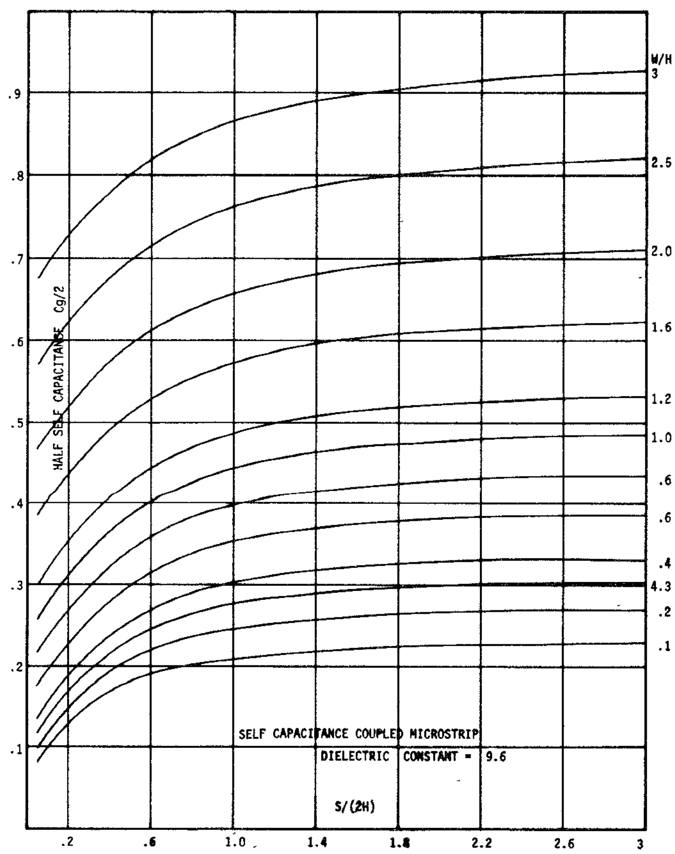

Fig. 3. Self-capacitance coupled microstrip dielectric constant of 9.6.

TABLE I
SELF-CAPACITANCE FOR UNCOUPLED MICROSTRIP
DIELECTRIC CONSTANT OF 9.6

W/H	$C_g/2$
3.0	.9510
2.5	.8409
2.0	.7293
1.6	.6386
1.2	.5456
1.0	.4979
.8	.4494
.6	.3969
.4	.3398
.3	.3078
.2	.2713
.1	.2245

TABLE II
DESIGN TABLE OF TRIAL FILTER

Abscissa Value	$C_{k, k+1}$
.0846	C_{01}
.0889	C_{01}
.9251	C_{12}
.9488	C_{12}
1.2209	C_{23}
1.2217	C_{23}
	C_{34}

Therefore,

$S_{01}/H = .0846 + .0889 = .1735 = S_{56}/H$

$S_{12}/H = .9251 + .9488 = 1.8739 = S_{45}/H$

$S_{23}/H = 1.2209 + 1.2217 = 2.4426 = S_{34}/H$

TABLE III
RENORMALIZED CAPACITANCES FOR A 10-PERCENT BANDWIDTH
INTERDIGITAL FILTER DESIGN WITH $N = 5$ RESONATORS

K	$C_{k, k+1}$	K	C_k
0 and 5	.265	0 and 6	.735
1 and 4	.0643	1 and 5	.770
2 and 3	.0490	2 and 4	.918
		3	.932

charts is the same as Cristal's charts for finding the dimensions of coupled lines and interdigital structures with one exception: The coupling in microstrip falls off so slowly that it is necessary to give $C_g/2$ for the uncoupled strip in Table I to be used instead of the end of the graph (Fig. 3) as is done by Cristal for the case of a strip coupled on one side.

The design of the trial filter on Teflon–Fiberglas is presented in Table II to illustrate the proper use of the graphs. Table III gives renormalized capacitances for the interdigital filter, which were obtained by dividing both the self and mutual capacitances obtained by the Matthaei [6] procedure by 7.534. The discussion of how to use the graphs will not be presented again here; see Cristal for a thorough explanation.

TABLE IV
INTERDIGITAL FILTER WITH 10-PERCENT BANDWIDTH AND
0.1-dB RIPPLE ON A 26-MIL SUBSTRATE OF DIELECTRIC CONSTANT 2.4

K	$C_{k, k+1}$	SPACING	K	C_k	LINE WIDTH
0 and 5	.265	4.5	0 and 6	.735	63.5
1 and 4	.0643	48.7	1 and 5	.770	75
2 and 3	.0490	63.5	2 and 4	.918	79.8
			3	.932	79.9

Table IV is a summary of the dimensions obtained for the trial filter. The procedure given by Matthaei [6] must be modified by adjusting the impedance level of the interior to 50 Ω instead of 70 Ω for coax construction. If this is not done, the interior lines will be thin and excessively lossy. This may be done by increasing the dimensionless parameter H.

III. POLYNOMIAL APPROXIMATIONS

So that this procedure could be programmed on a computer, polynomial approximations were developed for each of the curves on the graphs. These polynomials approximate the data in the Chebyshev sense, that is, the maximum error is minimized. The first set of approximations is

$$S/(2H) = \sum_{I=1}^{6} A_i (\ln (C_c))^{(i-1)} \tag{7}$$

for each W/H where CC is the mutual capacitance and $S/(2H)$ is one-half the edge spacing divided by the substrate thickness. The second set is

$$CG/2 = \sum_{I=1}^{6} B_i(S/(2H))^{(i-1)} \tag{8}$$

for each W/H. The coefficients A_i and B_i are given in Table V for a dielectric constant of 9.6. Table VI gives the set of coefficients for a dielectric constant of 2.40. For a dielectric constant of 2.4, it is sufficient to use a fourth-order approximation. The approximations are better in most cases than the curves and can be used to regenerate the curves if desired, but they are much more useful in a computer program. Table VII is a list of the $CG/2$ for various stripwidths to be used with uncoupled strips on a substrate of a dielectric constant of 2.40.

The polynomial approximations have been incorporated in a computer program to design interdigital structures and coupled lines. For each normalized mutual capacitance a set of points $(W/H, S/(2H))$ is found using coefficients A. Then for each $(W/H, S/(2H))$ the half-normalized self-capacitance is found using coefficients B. Starting with the first strip, the $CG/2$ for each $(W/H, S/(2H))$ corresponding to the first mutual capacitance is added to the corresponding $CG/2$ in Table I, self-capacitance of uncoupled strips. Quadratic interpolation is used to match the CG for the required self-capacitance giving the width of the first strip. The same quadratic interpolation routine is then applied to the set of numbers $(W/H, S/(2H))$ to get $S/(2H)$. Linear interpolation would be sufficient between the various W/H, but quadratic interpolation is recommended for the second interpolation.

For the second strip in an interdigital structure the polynomial approximations are applied to the second mutual capacitance, and again a set of points $(CG/2, W/H, S/(2H))$ is found. These new $CG/2$ are added to the $CG/2$ obtained from the

TABLE V
POLYNOMIAL APPROXIMATION COEFFICIENTS FOR DIELECTRIC CONSTANT OF 9.6

DIELECTRIC CONSTANT = 9.6

COEFFICIENTS A

ALOG(CC) TO S/(2*H)

W/H	COEFFICIENTS	2	3	4	5	6
3.00	.318040E-01	.202539E+00	.434785E+00	.147609E+00	.351423E-01	.217799E-02
2.50	.332510E-01	.188761E+00	.401058E+00	.128244E+00	.284548E-01	.157045E-02
2.00	.339473E-01	.172477E+00	.363217E+00	.107561E+00	.217437E-01	.992756E-03
1.60	.360997E-01	.165636E+00	.339676E+00	.957182E-01	.178383E-01	.692475E-03
1.20	.376814E-01	.156437E+00	.309918E+00	.810593E-01	.134787E-01	.376021E-03
1.00	.387925E-01	.152642E+00	.293476E+00	.730955E-01	.112628E-01	.226996E-03
.80	.405297E-01	.150355E+00	.275435E+00	.643457E-01	.895724E-02	.810432E-04
.60	.435612E-01	.150748E+00	.254333E+00	.539728E-01	.640606E-02	-.705746E-04
.40	.500234E-01	.157264E+00	.227842E+00	.406919E-01	.340521E-02	-.235281E-03
.30	.564917E-01	.165318E+00	.210863E+00	.320975E-01	.161082E-02	-.326128E-03
.20	.687306E-01	.180409E+00	.189245E+00	.211761E-01	-.512501E-03	-.424799E-03
.10	.100156E+00	.215192E+00	.159328E+00	.605381E-02	-.318002E-02	-.530562E-03

COEFFICIENTS B

S/(2*H) TO CG/2

W/H	COEFFICIENTS	2	3	4	5	6
3.00	.654978E+00	.426432E+00	-.323791E+00	.137691E+00	-.305920E-01	.275145E-02
2.50	.550681E+00	.431572E+00	-.331945E+00	.142023E+00	-.316700E-01	.285610E-02
2.00	.445972E+00	.437847E+00	-.343257E+00	.148846E+00	-.336221E-01	.307455E-02
1.60	.361855E+00	.443855E+00	-.355445E+00	.156621E+00	-.358797E-01	.332358E-02
1.20	.277600E+00	.449581E+00	-.369981E+00	.166576E+00	-.389279E-01	.367738E-02
1.00	.235539E+00	.451833E+00	-.378118E+00	.172349E+00	-.406687E-01	.387187E-02
.80	.192684E+00	.454363E+00	-.389824E+00	.181389E+00	-.435527E-01	.420715E-02
.60	.151895E+00	.457083E+00	-.409075E+00	.197957E+00	-.492102E-01	.489621E-02
.40	.112238E+00	.458513E+00	-.440662E+00	.228342E+00	-.602853E-01	.630744E-02
.30	.931251E-01	.458345E+00	-.465531E+00	.253728E+00	-.698214E-01	.754537E-02
.20	.748251E-01	.456483E+00	-.501217E+00	.291795E+00	-.844154E-01	.946312E-02
.10	.584293E-01	.445961E+00	-.549677E+00	.349206E+00	-.107329E+00	.125438E-01

first mutual capacitance for each W/H. The width of the second strip is obtained by interpolating this sum.

Using the first set (W/H, $S/(2H)$) and the second stripwidth, another $S/(2H)$ is found through interpolation. The sum of this $S/(2H)$ and the one interpolated from the set (W/H, $S/(2H)$) of the first mutual capacitance, i.e., between the first and second strips, is the edge spacing S/H between the first two strips. The $S/(2H)$ for the second spacing is interpolated from the second set (W/H, $S/(2H)$) using the second stripwidth. This procedure is continued through all the strips with the last strip handled the same as the first.

The accuracy of the approximations was checked by comparing the dimensions given by the program for coupled pairs of lines and the charts of Bryant and Weiss. The region $0.1 \leq S/H \leq 4.0$ and $0.1 \leq W/H \leq 2$ was checked. Over this region the maximum percent difference in the dimensions was 3.3 percent for S/H, but for most of the region the percent difference was less than 1 percent. The error rose for small values of S/H and W/H. For $W/H = 0.2$ the error in W/H was less than 2 percent. The approximations for W/H degraded for large spacings and small W/H. But even for $W/H = 0.1$ and $S/H = 4$, the error is only 10 percent which is of the same order as the tolerance on etching such narrow lines in thin films. For most practical problems this procedure of approximations gives results as accurate as the Bryant and Weiss data.

IV. TRIAL FILTER

The procedure will give reasonably accurate spacings and stripwidths for interdigital structures, but there are still problems which could not be answered easily analytically. The coupling between microstrip lines falls off quite slowly compared with stripline or rods between two ground planes, so that coupling between nonadjacent strips will exist. This coupling between nonadjacent strips is not accounted for in the equivalent circuits of the interdigital or combline filter. The second major problem is the electrical length of the resonators. The equivalent circuit of neither the interdigital nor the combline filter accurately takes care of the effect of the different velocities caused in the N-wire structure in an inhomogeneous medium. These problems cast serious doubts on the possibility of realizing interdigitated filters on microstrip.

To check the possibility of making interdigital filters on microstrip, a test filter was designed and fabricated on a 26-mil Teflon–Fiberglas substrate. The filter fabricated compares favorably with half-wave parallel coupled line bandpass filters realized on microstrip. The test filter is a five-section 10-percent bandwidth 0.1-dB ripple centered in the upper S band. In a two-wire interdigital structure, the stubs in the equivalent circuit are related to the even mode impedance only. The lines were all nearly the same width and the even mode velocity of a two-line coupled pair does not depend strongly on the spacing, so the same length was picked for all of them. The center frequency was close and only the middle resonator had to be shortened a little to give good return loss in the passband. Fig. 4 is a plot of the measured results of the test filter. This plot is similar to the curves for coupled line bandpass filters on microstrip. Both have low maximum rejection of about 40 dB. The only successful method to realize greater rejection is to build two filters and separate the two into closed boxes. The unloaded Q was originally estimated to be about 300 but it appears 200 is a closer value for this substrate. Even though the shape of the rejection bands does not follow the theoretical curve, the filter is useful because it has a flat response and good return loss in the passband.

TABLE VI
POLYNOMIAL APPROXIMATION COEFFICIENTS FOR DIELECTRIC CONSTANT OF 2.4

COUPLED MICROSTRIP

DIELECTRIC CONSTANT = 2.4

COEFFICIENTS A

W/H	COEFFICIENTS	2	3	4	5
4.00	.143252E+00	.490689E+00	.427396E+00	.814121E-01	.128268E-01
3.50	.158810E+00	.506215E+00	.432766E+00	.833286E-01	.122890E-01
3.00	.173525E+00	.518919E+00	.435330E+00	.845016E-01	.116856E-01
2.50	.186458E+00	.526742E+00	.433486E+00	.845005E-01	.109755E-01
2.00	.198618E+00	.530962E+00	.427597E+00	.834237E-01	.101688E-01
1.60	.199696E+00	.516954E+00	.410313E+00	.792974E-01	.922690E-01
1.20	.201435E+00	.501573E+00	.389686E+00	.743118E-01	.821292E-02
1.00	.203911E+00	.495054E+00	.378376E+00	.715167E-01	.767834E-02
.80	.209630E+00	.492114E+00	.367068E+00	.685917E-01	.712400E-02
.60	.222048E+00	.496641E+00	.356245E+00	.654790E-01	.653293E-02
.40	.249698E+00	.517410E+00	.346827E+00	.620306E-01	.586970E-02
.20	.330722E+00	.594241E+00	.347788E+00	.591291E-01	.513116E-02

COEFFICIENTS B

W/H	COEFFICIENTS	2	3	4	5
4.00	.437352E+00	.203505E+00	-.119052E+00	.348614E-01	-.397580E-02
3.50	.385396E+00	.205273E+00	-.121393E+00	.357162E-01	-.408283E-02
3.00	.333328E+00	.207245E+00	-.124200E+00	.367771E-01	-.421868E-02
2.50	.281110E+00	.209492E+00	-.127661E+00	.381335E-01	-.439646E-02
2.00	.228678E+00	.212104E+00	-.132075E+00	.399346E-01	-.463870E-02
1.60	.186540E+00	.214498E+00	-.136605E+00	.418653E-01	-.490534E-02
1.20	.144230E+00	.217010E+00	-.142431E+00	.445058E-01	-.528401E-02
1.00	.123056E+00	.218163E+00	-.146057E+00	.462592E-01	-.554472E-02
.80	.101955E+00	.219312E+00	-.150891E+00	.487509E-01	-.592961E-02
.60	.810060E-01	.218993E+00	-.155371E+00	.512868E-01	-.633331E-02
.40	.604816E-01	.217430E+00	-.162105E+00	.554877E-01	-.703428E-02
.20	.407921E-01	.211099E+00	-.171807E+00	.626854E-01	-.831606E-02

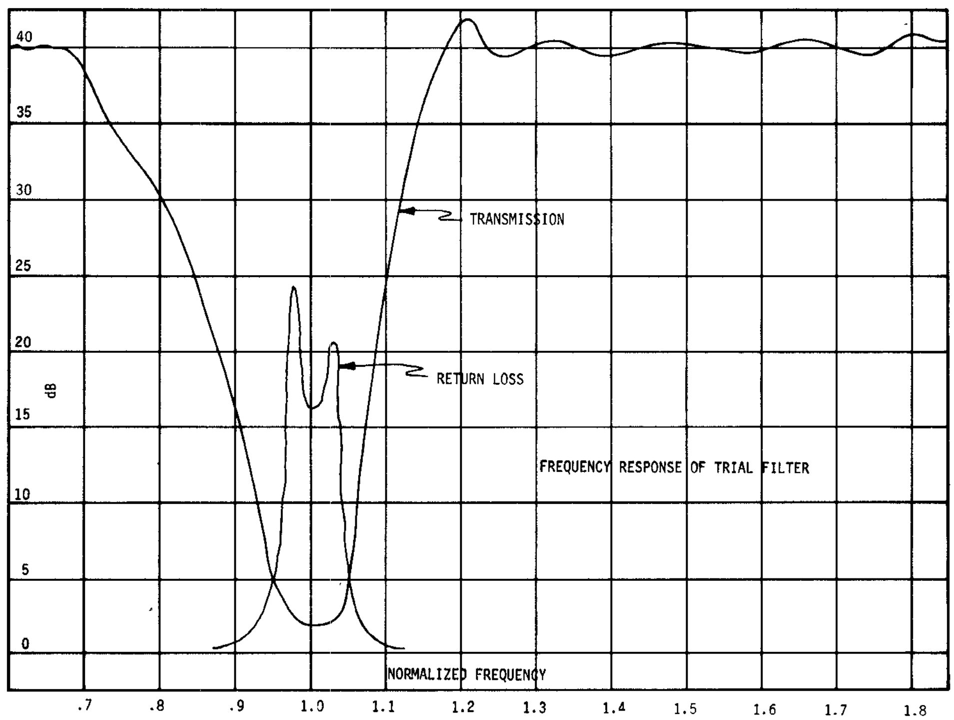

Fig. 4. Frequency response of trial filter.

The design bandwidth of the trial filter was 10 percent but the bandwidth of the filter, measured on a return loss basis, is only 6.2 percent at the 1.35 VSWR (0.1-dB ripple) points. The bandwidth of the filter measures 8.4 percent based on the 3-dB bandwidth and referenced back to the 0.1-dB points. Also for a five-section filter there should be five points of match, but the response only has two. The stopband's attenuation does not correspond with theory at all. In the author's experience, half-wave parallel coupled line bandpass filters also exhibit similar deviations from theory when fabricated on microstrip.

The interdigital filter is superior to the half-wave parallel coupled line bandpass filter on microstrip because the interdigital

TABLE VII
SELF-CAPACITANCES FOR UNCOUPLED MICROSTRIP DIELECTRIC CONSTANT OF 2.4

W/H	CG/2
4.0	.6147
3.50	.5596
3.0	.5040
2.5	.4477
2.0	.3904
1.6	.3436
1.2	.2954
1.0	.2705
.8	.2446
.6	.2173
.4	.1874
.2	.1514

microstrip filter does not have a spurious second harmonic response, and it takes up less space at the expense of shorts through the substrate. Microstrip filters are not viable by themselves because they have poor ultimate rejection, high loss, and do not follow the theoretical curves; but it is sometimes convenient and economical to use them.

V. CONCLUSION

In conclusion, a procedure is given which can be used to design coupled line and interdigital structures on microstrip. The procedure can be easily computer programmed using the polynomial approximations to give accurate results with very short computation times compared with times required by the Bryant and Weiss method or even Smith's approximations. Most importantly, since this procedure leads from electrical parameters to dimensions, it can be incorporated in automatic design programs.

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Asymmetric Even-Mode Fringing Capacitance

HENRY J. RIBLET, FELLOW, IEEE

***Abstract*—An expression is given for the even-mode fringing capacitance of an infinite rectangular bar, asymmetrically located inside an infinite u-shaped outer conductor.**

Manuscript received May 3, 1976; revised August 30, 1976.
The author is with the Microwave Development Laboratories, Inc., Needham Heights, MA 02194.

INTRODUCTION

The writer [1] has recently given a closed expression for the odd-mode fringing capacitance for an infinite rectangular bar, asymmetrically located in an infinite u-shaped outer conductor. The essential problem solved in that note was the determination of the conformal transformation which maps the upper half t plane into the doubly infinite u-shaped polygon in the z plane as shown in Fig. 1. The determination of the capacitance of the structure presented no problem since it could be found from well-known formulas with the help of the "excess capacitance" introduced by Riblet [2].

If the even-mode capacitance is defined in a manner consistent with that used by Getsinger [3], as the capacitance of the structure in the z plane when the line segment BC is a magnetic wall, then we require in the t plane the capacitance of two separated line segments, AB and CD, both at the same potential, with respect to the infinite line segment DA. The determination of the limiting value of this capacitance is the essential problem of this short paper.

THE EVEN-MODE CAPACITANCE

In the t plane, the capacitance between the two-line segments, $[\mu + \delta\mu, 1]$ and $[1/k^2, \nu - \delta\nu]$ maintained at the same potential, and the infinite line segment $[\nu + \delta\nu, \mu - \delta\mu]$ is required in the limit as $\delta\mu$ and $\delta\nu \to 0$. It is important to keep in mind that the small semicircles about A and D are magnetic walls, while the semicircles about O, B, C, and E play no essential role in the calculations. This capacitance is not altered if the upper half of the t plane is mapped onto the upper half of the s plane so that B maps into -1, C into $+1$, A into $-l$, and D into $+l$. This is accomplished by the linear transformation

$$s = \gamma \frac{t - \alpha}{t - \beta} \tag{1}$$

if α, β, and γ are selected so that

$$\gamma \frac{1/k^2 - \alpha}{1/k^2 - \beta} = -\gamma \frac{1 - \alpha}{1 - \beta} = 1 \tag{2}$$

and

$$\gamma \frac{\nu - \alpha}{\nu - \beta} = -\gamma \frac{\mu - \alpha}{\mu - \beta} = l. \tag{3}$$

Again it is important that the semicircles about $-l$ and $+l$ and the line segment between B and C be magnetic walls. From (2) and (3)

$$\begin{aligned} 2\alpha\beta - (1 + 1/k^2)(\alpha + \beta) + 2/k^2 &= 0 \\ 2\alpha\beta - (\mu + \nu)(\alpha + \beta) + 2\mu\nu &= 0. \end{aligned} \tag{4}$$

Whenever $\mu + \gamma \neq 1 + 1/k^2$, this set of equations can be solved uniquely for $\alpha\beta$ and $\alpha + \beta$. It is then a simple matter to solve the quadratic equation

$$\chi^2 - (\alpha + \beta)\chi + \alpha\beta = 0$$

to determine α and β. Gamma is then found from either (2) or (3). The total capacitance of the system is unchanged by the transformation, and, if we take the radii of the semicircles about A and D in the s plane to be the same, the geometry and the lines of force in the s plane are completely symmetrical about the imaginary axis. Thus it may be replaced by a magnetic wall. Then one-half of the limiting value of the total capacitance of the system is given by the limiting value of the capacitance of the finite line segment, $[1, l - \delta s]$, with respect to the infinite